Deformations of 4d SCFTs and Supersymmetry Enhancing RG Flows

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Introduction

Symmetry is one of the most important quantities which partly characterizes QFT.

We usually define a theory in UV and analyze the RG flow and its IR theory.

(Suppose we have a nontrivial fixed point in IR, then)
Does the symmetry in UV still characterize the IR theory? Or is the IR symmetry same as the UV symmetry?

The IR symmetry could be different from the UV symmetry.

Susy enhancement

We consider enhancement of supersymmetry in 4d supersymmetric QFTs along a renormalization group flow.

Few example is known for supersymmetry in 4d:

- N=2 conformal SU(n) SQCD (with gauge coupling g), then change the superpotential coupling to generic value $W = h\, q\, \Phi\, q' \rightarrow$ N=2
- N=1 Lagrangian theories where a coupling constant is set to infinity → N=2 E_6, E_7 and $R_{0,N}$ theories **[Gadde-Razamat-Willet, Agarwal-KM-Song]**

N=1 SU(2) gauge theory with **[KM-Song]**

- two fundamental chirals q, q'
- adjoint chiral ϕ
- two singlet chirals X, M

	q	q'	ϕ	M	X
$U(1)_{R0}$	1/2	-5/2	1	6	0
$U(1)_{\mathcal{F}}$	1/2	7/2	-1	-6	2
$U(1)_R$	14/15	8/15	2/15	4/5	26/15

with superpotential

$$W = X\mathrm{tr}\phi^2 + \mathrm{tr}\phi q^2 + M\mathrm{tr}\phi q'^2$$

By a-maximization, we get the central charges

$$a = \frac{43}{120}, \quad c = \frac{11}{30}, \quad \Delta(M) = \frac{6}{5}$$

which are the same as those of **Argyres-Douglas theory H_0** (an N=2 superconformal field theory (SCFT)).

- By checking the superconformal index, one can show that there is indeed an N=2 supersymmetry.
- Thus, it's likely that the Argyres-Douglas theory is realized at this fixed point.

The Argyres-Douglas theory

- was originally found at a special point on the Coulomb branch of N=2 SU(3) pure SYM with mutually non-local massless particles

[Argyres-Douglas, Argyres-Plesser-Seiberg-Witten]

- There is no weak-coupling cusp (no exactly marginal coupling) and the Coulomb branch operator has scaling dimension 6/5
- **The UV Lagrangian theory can be used to compute partition functions, e.g. superconformal index**

Questions:

- Mechanism of the susy enhancement?
- How widely does this enhancement happen?

The coupling with (gauge-)singlet chiral is a key point.
This has not been fully studied so far, and could lead to an IR fixed point with enhanced symmetry **[Seiberg's dual theory, Kim-Razamat-Vafa-Zafrir]**

In this talk, we will see two methods, which accommodate such kind of coupling, and see the enhancement is general phenomenon:

- Nilpotent deformations of N=2 SCFTs with non-Abelian flavor symmetry
- Systematic deformation of N=1 SCFTs

N=1 deformation

Suppose we have an N=2 SCFT ***T*** with **non-Abelian flavor symmetry F.**

[Gadde-KM-Tachikawa-Yan, Agarwal-Bah-KM-Song]
[Agarwal-Intriligator-Song]
cf. [Heckman-Tachikawa-Vafa-Wecht]

Then let us

- **couple N=1 chiral multiplet M in the adjoint rep of F by the superpotential**

$$W = \mathrm{tr}\mu M$$

- **give a nilpotent vev to M** (which is specified by the embedding ρ: SU(2)→F)**, which breaks F** $W = \sum_j \mu_{j,j} M_{j,-j}$

(For F=SU(N), this is classified by a partition of N or Young diagram.)

This gives IR theory *T*$_{\mathbf{IR}}$[*T*, ρ], which is generically N=1 supersymmetric.

Conditions for "N=2"

For principal embedding: we conjecture that the condition for *T* to have enhancement of supersymmetry in the IR is as follows:

- F is of ADE type
- 2d chiral algebra stress-tensor is the Sugawara stress-tensor:

[Beem-Lemos-Liendo-Peelaers-Rastelli-van Rees]

$$\frac{\dim F}{c} = \frac{24h^\vee}{k_F} - 12$$

- **rank-one theories H_1, H_2, D_4, E_6, E_7, E_8 → H_0**
- **SU(N) SQCD with 2N flavors → (A_1, A_{2N})**
- **Sp(N) SQCD with 2N+2 flavors → (A_1, A_{2N+1})**
- **(A_1, D_k) theory → (A_1, A_{k-1})**
- **some quiver gauge theories → (A_N, A_L)**

[Agarwal-Sciarappa-Song, Benvenuti-Giacomelli]

T = SU(2) w/ 4 flavors

In this case, F = SO(8)

We consider the principal embedding of SO(8), the vev which breaks SO(8) completely.

The adjoint rep decomposes as

28 → 3, 7, 7, 11

$$M_{1,-1}, M_{3,-3}, M'_{3,-3}, M_{5,-5}$$

→ after integrating out the massive fields, we get SU(2) w/ 1 flavor and adjoint and the superpotential

$$W = \mathrm{tr}\phi q^2 + M_5 \mathrm{tr}\phi q'^2$$

Central charges

The central charges of the SCFT are determined from the anomaly coefficients of the IR R-symmetry: **[Anselmi-Freedman-Grisaru-Johansen]**

$$a = \frac{3}{32}(3\mathrm{Tr}R_{\mathrm{IR}}^3 - \mathrm{Tr}R_{\mathrm{IR}}), \quad c = \frac{1}{32}(9\mathrm{Tr}R_{\mathrm{IR}}^3 - 5\mathrm{Tr}R_{\mathrm{IR}})$$

In our case, the IR R-symmetry is a combination of two U(1)'s. Thus consider the following

$$R_{\mathrm{IR}}(\epsilon) = R_0 + \epsilon\mathcal{F}$$

The true R symmetry is determined by maximizing trial central charge [Intriligator-Wecht]

$$a(\epsilon) = \frac{3}{32}(3\mathrm{Tr}R_{\mathrm{IR}}(\epsilon)^3 - \mathrm{Tr}R_{\mathrm{IR}}(\epsilon))$$

Decoupling issue

The trϕ^2 operator hits the unitarity bound (Δ<1). We interpret this as being decoupled. Thus we subtract its contribution from central charge, and re-a-maximize

[cf. Kutasov-Parnachev-Sahakyan]

$$\mathrm{Tr}\phi^2, \ M, \ \ldots$$

dimension 6/5

$$\epsilon = \frac{13}{15}, \quad a = \frac{43}{120}, \quad c = \frac{11}{30}$$

A way to pick up the interacting part is by introducing a chiral multiplet X to set trϕ^2=0: $\delta W = X\mathrm{tr}\phi^2$

$$a_{\mathrm{chiral}}(r) = -a_{\mathrm{chiral}}(2-r)$$

In the end, the Lagrangian which flows to the Argyres-Douglas theory (H_0 theory) is

$$W = \mathrm{tr}\phi q^2 + M\mathrm{tr}\phi q'^2 + X\mathrm{tr}\phi^2$$

Chiral ring of H_0

We had the following chiral operators

$$\mathrm{tr}\phi q^2, \quad \mathrm{tr}\phi q q', \quad \mathrm{tr} q q', \quad \mathrm{tr}\phi q'^2, \quad X, \quad M$$

The F-term conditions are

$$0 = qq + Mq'^2 + 2X\phi, \quad 0 = \mathrm{tr}\phi q'^2, \quad 0 = \phi q, \quad 0 = M\phi q', \quad 0 = \mathrm{tr}\phi^2.$$

Thus, **the generators in the chiral ring** are only

$$\mathrm{tr} q q', \quad M$$

(moduli space of X is uplifted quantum mechanically)

dim = 11/5, 6/5 → **form N=2 Coulomb branch operator multiplet**

T = SU(2) w/ 4 flavors

Other choices of embeddings:

- **[5,1³], [4,4]** (with SU(2)) → **H_1 theory** (SU(2) flavor symmetry)

$$a = \frac{11}{24}, \quad c = \frac{1}{2}$$

- **[3²,1²]** (with U(1)xU(1)) → **H_2 theory** (SU(3) flavor symmetry)

$$a = \frac{7}{12}, \quad c = \frac{2}{3}$$

- other embeddings → N=1 SCFTs

H_1 theory

By the deformation procedure one can obtain **SU(2) gauge theory with the following chiral multiplets**:

	(q, q')	ϕ	M	X
SU(2)	**2**	**adj**	**1**	**1**
$U(1)_{R0}$	-1	1	4	0
$U(1)_{\mathcal{F}}$	2	-1	-4	2
$SU(2)_f$	**2**	**1**	**1**	**1**

with the superpotential

$$W = X\mathrm{tr}\phi^2 + Mqq'$$

This theory flows to the H_1 theory with central charges

$$a = \frac{11}{24},\ c = \frac{1}{2}$$

N=2? on Coulomb branch

From the Argyres-Douglas theory viewpoint, one can go to the Coulomb branch by turning on

- vev of Coulomb branch operator $\langle \mathcal{O} \rangle = u$
- relevant coupling: $\delta \mathscr{L} = c \int d^2\theta_1 d^2\theta_2 U$
- mass deformation: $\delta \mathscr{L} = m \int d^2\theta_1 \mu_0, \quad (\mu_0 : \text{moment map operator})$

One can study the physics on the IR Coulomb branch from the Lagrangian viewpoint: for the H_1 theory, the above deformations correspond to adding

$$W = X\mathrm{tr}\phi^2 + uqq' + cX + m\mathrm{tr}\phi qq'$$

The theory with superpotential

$$W = uqq' + m\phi qq'$$

has been studied by **[Intriligator-Seiberg]**. They found the theory is in **N=1 Coulomb branch** parametrized by $v = \langle \mathrm{tr}\phi^2 \rangle$, whose curve is given by

$$y^2 = x^3 - vx^2 + \frac{1}{4}u\Lambda^3 x - \frac{1}{64}m^2\Lambda^6$$

Adding the terms $X\phi^2 + cX$ sets the vev $v = \langle \mathrm{tr}\phi^2 \rangle$ to -c. Thus the N=1 curve is now

$$y^2 = x^3 + cx^2 + \frac{1}{4}u\Lambda^3 x - \frac{1}{64}m^2\Lambda^6$$

which is indeed the same as the Seiberg-Witten curve of the N=2 H_1 theory after the redefinition of the parameters.

Superconformal index

Now we had Lagrangian theories which flow to SCFTs in the IR. **Thus the superconformal indices of the latter can be simply computed from the matter content.**

The index of our N=1 theory is defined by

[Kinney-Maldacena-Minwalla-Raju, Romelsberger]

$$\mathcal{I} = \mathrm{Tr}_{\mathcal{H}_{S^3}} (-1)^F p^{j_1+j_2-R/2} q^{j_2-j_1-R/2} \prod_i a_i^{F_i}$$
$$= \mathrm{Tr}_{\mathcal{H}_{S^3}} (-1)^F t^{3(R+2j_1)} y^{2j_2} \prod_i a_i^{F_i} \qquad (p = t^3 y, \quad q = t^3/y)$$

where j_1 and j_2 are rotation generators of the maximal torus $U(1)_1$ and $U(1)_2$ of $SO(4)=SU(2)_1 \times SU(2)_2$ and R and *Fi* is the generators of the $U(1)_R$ and Cartans of flavor symmetry.

(If S^3 is described by equation $|x_1|^2+|x_2|^2=1$, j_1+j_2 and j_1-j_2 rotate x_1 and x_2 by phase.)

Index of H0 theory

For instance one could calculate the index of the Argyres-Douglas (H_0) theory from the Lagrangian:

$$I = \kappa \frac{\Gamma((pq)^3 \xi^{-6})}{\Gamma((pq)^1 \xi^{-2})} \oint \frac{dz}{2\pi i z} \frac{\Gamma(z^{\pm}(pq)^{\frac{1}{4}}\xi^{\frac{1}{2}})\Gamma(z^{\pm}(pq)^{-\frac{5}{4}}\xi^{\frac{7}{2}})\Gamma(z^{\pm 2,0}(pq)^{\frac{1}{2}}\xi^{-1})}{\Gamma(z^{\pm 2})}$$

ξ : fugacity for $\mathrm{U}(1)_{\mathcal{F}}$

(We subtract the contributions of the decoupled operators!)

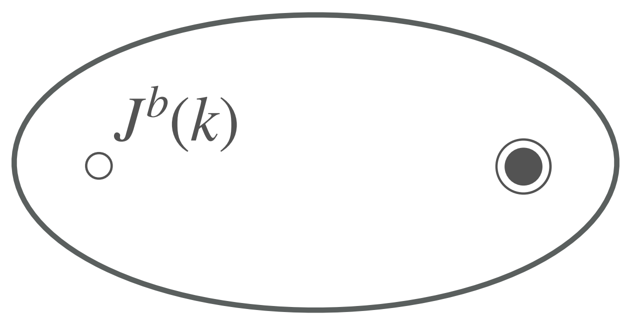
We substitute $\xi \to t^{\frac{1}{5}}(pq)^{\frac{3}{10}}$ for the correct IR R symmetry. After that

- basically one can compute the integral
- Coulomb index limit (pq/t=u, p,q,t→0): $I_C = \frac{1}{1-u^{\frac{6}{5}}}$
- Macdonald limit (p→0) agrees with the index by **[Cordova-Shao, Song]**

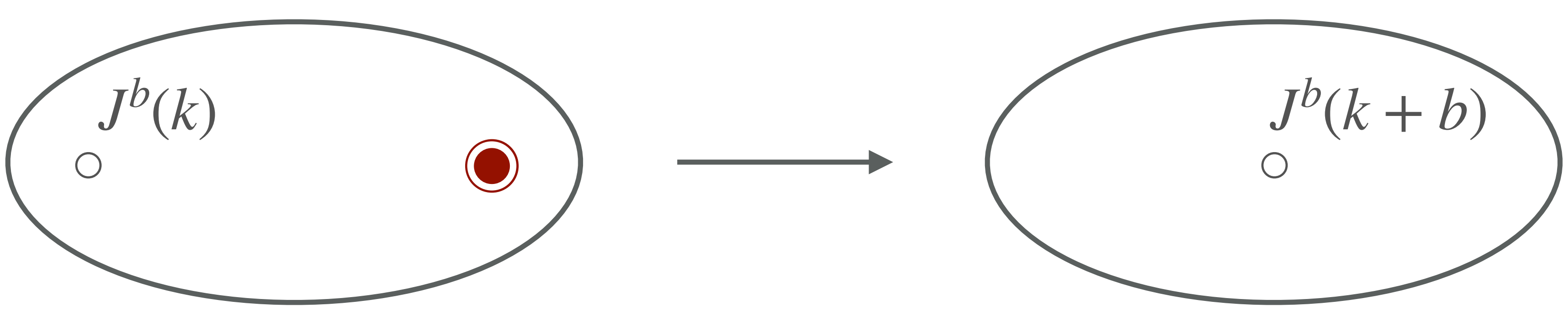
Class S interpretation

All the theories T, which show the IR enhancement of supersymmetry by nilpotent principal deformation, are of class S **[Gaiotto]**, in terms of a sphere with one irregular and one regular punctures:

$$j^b(k) : \phi_{\text{Hitchin}}(z) \sim \frac{A}{(z-z_0)^{2+k/b}} + \dots$$



The nilpotent deformation above is done by changing the twisting (N=1 twist) **[Bah-Beem-Bobev-Wecht]** and by closing the regular puncture **[Gadde-KM-Tachikawa-Yan]**



[Giacomelli]

General deformations of N=1 SCFTs

Systematic deformation procedure

[Nardoni-KM-Song]

1. **Suppose we have an N=1 SCFT, $T_{N=1}$**

2. **find all the relevant operators $\mathcal{O}$ (R < 2) and all the "super"-relevant operator $\mathcal{O}_s$ (R < 4/3)**

3. **deform SCFT by each relevant operator, or by each super-relevant operator by coupling with free chiral multiplet M:**

$$\int d^2\theta \mathcal{O} M$$

4. at each fixed point, return to 2 and repeat the procedure, and stop if it terminates

- **<u>For step 2</u>, it is enough to know the superconformal index for the purpose to find the relevant operators.**

- **Once we could get the index it is convenient to consider the "reduced" index and the expansion in the variable t.**

$$\mathcal{I}_{\text{red}} = (1 - t^3 y)(1 - t^3 y^{-1})(\mathcal{I} - 1)$$

example H_0:

$$\mathcal{I}_{\text{red}} = t^{\frac{12}{5}} v^{\frac{6}{5}} - t^{\frac{17}{5}} v^{\frac{1}{5}} \chi_2(y) + t^{\frac{22}{5}} v^{-\frac{4}{5}} + t^{\frac{24}{5}} v^{\frac{12}{5}} - t^{\frac{29}{5}} v^{\frac{7}{5}} \chi_2(y) - t^6 + \ldots$$

- **<u>For Step 3</u>, one can find the fixed point by a-maximization.**

- **The index of the fixed point can be obtained by setting the flavor fugacities according to the mixing, then we return to point 2**

- The index cannot have the terms which indicating the unitarity-violation. If there is no such term, we call the fixed points as "good".

- **Results for simple SCFTs:** $T_{N=1}$ = the fixed point of

  adjoint SU(2) w/ N_f=1 34 good fixed points; **N=2 H_0 and H_1**

  adjoint SU(3) w/ N_f=1 41 good fixed points; **N=2 (A_1, A_5)**

  adjoint SU(2) w/ N_f=2 ??? fixed points; **N=2 H_0, H_1 and H_2**

- Duality of theories adjoint SU(2) w/ N_f=1 and N_f=2 (whose fixed point is H_1 theory).

For $T_{N=1}$ = (the fixed point of **adjoint SU(2) w/ N_f=1**)

	(a,c)	$R(q)$	$R(\tilde{q})$	$R(\phi)$	$R(X_i)$	$R(M_i)$
1	$\left(\frac{263}{768},\frac{271}{768}\right)\simeq(0.3424,0.3529)$	$\frac{11}{12}$	$\frac{5}{12}$	$\frac{1}{6}$	$\frac{5}{3}$	1
2	$\left(\frac{1465\sqrt{1465}+81108}{397488},\frac{1051\sqrt{1465}+29088}{198744}\right)\simeq(0.3451,0.3488)$	$\frac{543-\sqrt{1465}}{546}$	$\frac{75-\sqrt{1465}}{78}$	$\frac{\sqrt{1465}+3}{273}$	$\frac{2\left(270-\sqrt{1465}\right)}{273}$	
3	$\left(\frac{711}{2048},\frac{807}{2048}\right)\simeq(0.3472,0.3940)$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{2}$	$\frac{5}{4},\frac{3}{4}$
4	$\left(\frac{43}{120},\frac{11}{30}\right)\simeq(0.3583,0.3667)$	$\frac{8}{15}$	$\frac{14}{15}$	$\frac{2}{15}$	$\frac{26}{15}$	$\frac{4}{5}$
5	$\left(\frac{375}{1024},\frac{439}{1024}\right)\simeq(0.3662,0.4287)$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{2}$	$\frac{5}{4},\frac{3}{4},\frac{3}{4}$
6	$\left(\frac{2211}{5488},\frac{1277}{2744}\right)\simeq(0.4029,0.4654)$	$\frac{4}{7}$	$\frac{2}{7}$	$\frac{2}{7}$	$\frac{10}{7}$	$\frac{8}{7},\frac{6}{7}$
7	$\left(\frac{14535}{35152},\frac{8535}{17576}\right)\simeq(0.4135,0.4856)$	$\frac{6}{13}$	$\frac{4}{13}$	$\frac{4}{13}$	$\frac{18}{13}$	$\frac{14}{13},\frac{12}{13},\frac{14}{13},\frac{12}{13}$
8	$\left(\frac{7441\sqrt{7441}+628560}{3072432},\frac{4606\sqrt{7441}+348435}{1536216}\right)\simeq(0.4135,0.4854)$	$\frac{783-5\sqrt{7441}}{759}$	$\frac{147+\sqrt{7441}}{759}$	$\frac{147+\sqrt{7441}}{759}$	$\frac{2\left(612-\sqrt{7441}\right)}{759}$	$\frac{359-\sqrt{7441}}{253},\frac{147+\sqrt{7441}}{253}$
9	$\left(\frac{285}{686},\frac{167}{343}\right)\simeq(0.4155,0.4869)$	$\frac{4}{7}$	$\frac{2}{7}$	$\frac{2}{7}$	$\frac{10}{7}$	$\frac{8}{7},\frac{6}{7},\frac{6}{7}$
10	$\left(\frac{924}{2197},\frac{1093}{2197}\right)\simeq(0.4206,0.4975)$	$\frac{4}{13}$	$\frac{6}{13}$	$\frac{4}{13}$	$\frac{18}{13}$	$\frac{10}{13},\frac{12}{13},\frac{14}{13},\frac{16}{13},\frac{12}{13}$
11	$\left(\frac{4\left(896\sqrt{7}+1665\right)}{38307},\frac{4036\sqrt{7}+8355}{38307}\right)\simeq(0.4214,0.4969)$	$\frac{378-80\sqrt{7}}{339}$	$\frac{4\left(4\sqrt{7}+15\right)}{339}$	$\frac{4\left(4\sqrt{7}+15\right)}{339}$	$\frac{-2\left(16\sqrt{7}-279\right)}{339}$	$\frac{-2\left(8\sqrt{7}-83\right)}{113},\frac{4\left(4\sqrt{7}+15\right)}{113},\frac{4\left(4\sqrt{7}+15\right)}{113}$
12	$\left(\frac{7587}{17576},\frac{2277}{4394}\right)\simeq(0.4317,0.5182)$	$\frac{6}{13}$	$\frac{4}{13}$	$\frac{4}{13}$	$\frac{18}{13}$	$\frac{14}{13},\frac{12}{13},\frac{14}{13},\frac{10}{13},\frac{12}{13}$
13	$\left(\frac{339}{784},\frac{97}{196}\right)\simeq(0.4324,0.4949)$	$\frac{1}{2}$	$\frac{5}{14}$	$\frac{2}{7}$	$\frac{10}{7}$	$1,\frac{6}{7},\frac{8}{7}$
14	$\left(\frac{5665\sqrt{5665}+162189}{1359456},\frac{5903\sqrt{5665}+262863}{1359456}\right)\simeq(0.4329,0.5202)$	$\frac{5\sqrt{5665}-27}{714}$	$\frac{291-\sqrt{5665}}{714}$	$\frac{291-\sqrt{5665}}{714}$	$\frac{\sqrt{5665}+423}{357}$	$\frac{\sqrt{5665}+185}{238},\frac{291-\sqrt{5665}}{238},\frac{397-3\sqrt{5665}}{238}$
15	$\left(\frac{15423}{35152},\frac{9317}{17576}\right)\simeq(0.4388,0.5301)$	$\frac{4}{13}$	$\frac{6}{13}$	$\frac{4}{13}$	$\frac{18}{13}$	$\frac{10}{13},\frac{12}{13},\frac{14}{13},\frac{16}{13},\frac{12}{13},\frac{10}{13}$

16	$\left(\frac{24817\sqrt{24817}+1456776}{12144432}, \frac{13666\sqrt{24817}+1101111}{6072216}\right)$ $\simeq (0.4419, 0.5359)$	$\frac{5\sqrt{24817}-27}{1509}$	$\frac{609-\sqrt{24817}}{1509}$	$\frac{609-\sqrt{24817}}{1509}$	$\frac{2\left(\sqrt{24817}+900\right)}{1509}$	$\frac{\sqrt{24817}+397}{503}, \frac{609-\sqrt{24817}}{503}, \frac{609-\sqrt{24817}}{503}, \frac{821-3\sqrt{24817}}{503}$
17	$\left(\frac{1221}{2744}, \frac{1417}{2744}\right) \simeq (0.4450, 0.5164)$	$\frac{1}{2}$	$\frac{5}{14}$	$\frac{2}{7}$	$\frac{10}{7}$	$1, \frac{6}{7}, \frac{8}{7}, \frac{6}{7}$
18	$\left(\frac{97\sqrt{97}+423}{3072}, \frac{113\sqrt{97}+471}{3072}\right) \simeq (0.4487, 0.5156)$	$\frac{123-7\sqrt{97}}{96}$	$\frac{45-\sqrt{97}}{96}$	$\frac{\sqrt{97}+3}{48}$	$\frac{45-\sqrt{97}}{24}$	$1, \frac{\sqrt{97}+3}{16}$
19	$\left(\frac{19\sqrt{19}-72}{24}, \frac{5(4\sqrt{19}-15)}{24}\right) \simeq (0.4508, 0.5074)$	$\frac{7-\sqrt{19}}{4}$	$\frac{27-5\sqrt{19}}{12}$	$\frac{\sqrt{19}-3}{6}$	$\frac{9-\sqrt{19}}{3}$	$\frac{2\left(\sqrt{19}-3\right)}{3}, \frac{2\left(6-\sqrt{19}\right)}{3}, \frac{\sqrt{19}-3}{2}$
20	$\left(\frac{621}{1372}, \frac{2925}{5488}\right) \simeq (0.4526, 0.5330)$	$\frac{1}{2}$	$\frac{5}{14}$	$\frac{2}{7}$	$\frac{10}{7}$	$1, \frac{6}{7}, \frac{8}{7}, \frac{5}{7}$
21	$\left(\frac{927}{2048}, \frac{1023}{2048}\right) \simeq (0.4526, 0.4995)$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{2}$	1
22	$\left(\frac{601\sqrt{601}+15012}{65712}, \frac{430\sqrt{601}+5841}{32856}\right) \simeq (0.4527, 0.4986)$	$\frac{105-2\sqrt{601}}{111}$	$\frac{105-2\sqrt{601}}{111}$	$\frac{\sqrt{601}+3}{111}$	$\frac{-2\left(\sqrt{601}-108\right)}{111}$	
23	$\left(\frac{11}{24}, \frac{1}{2}\right) \simeq (0.4583, 0.5000)$	$\frac{5}{9}$	$\frac{5}{9}$	$\frac{2}{9}$	$\frac{14}{9}$	$\frac{8}{9}$
24	$\left(\frac{2553}{5488}, \frac{3043}{5488}\right) \simeq (0.4652, 0.5545)$	$\frac{1}{2}$	$\frac{5}{14}$	$\frac{2}{7}$	$\frac{10}{7}$	$1, \frac{6}{7}, \frac{8}{7}, \frac{5}{7}, \frac{6}{7}$
25	$\left(\frac{483}{1024}, \frac{547}{1024}\right) \simeq (0.4717, 0.5342)$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{2}$	$1, \frac{3}{4}$
26	$\left(\frac{352\sqrt{22}+1251}{6144}, \frac{416\sqrt{22}+1347}{6144}\right) \simeq (0.4723, 0.5368)$	$\frac{2\sqrt{22}+3}{24}$	$\frac{21-2\sqrt{22}}{24}$	$\frac{1}{4}$	$\frac{3}{2}$	$1, \frac{9-\sqrt{22}}{6}$
27	$\left(\frac{61\sqrt{61}-441}{75}, \frac{127\sqrt{61}-912}{150}\right) \simeq (0.4723, 0.5327)$	$\frac{39-4\sqrt{61}}{15}$	$\frac{39-4\sqrt{61}}{15}$	$\frac{2\left(\sqrt{61}-6\right)}{15}$	$\frac{2\left(27-2\sqrt{61}\right)}{15}$	$\frac{2\left(\sqrt{61}-6\right)}{5}$
28	$(0.4727, 0.5351)$	0.5258	0.5009	0.2433	1.513	0.7051
29	$\left(\frac{1005}{2048}, \frac{1165}{2048}\right) \simeq (0.4907, 0.5688)$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{2}$	$1, \frac{3}{4}, \frac{3}{4}$
30	$\left(\frac{44\sqrt{22}+171}{768}, \frac{13\left(4\sqrt{22}+15\right)}{768}\right) \simeq (0.4914, 0.5715)$	$\frac{2\sqrt{22}+3}{24}$	$\frac{21-2\sqrt{22}}{24}$	$\frac{1}{4}$	$\frac{3}{2}$	$1, \frac{3}{4}, \frac{9-\sqrt{22}}{6}$
31	$\left(\frac{89\sqrt{\frac{89}{17}}-180}{48}, \frac{44\sqrt{\frac{89}{17}}-87}{24}\right) \simeq (0.4925, 0.5698)$	$\frac{2\sqrt{\frac{89}{17}}-3}{3}$	$\frac{2\sqrt{\frac{89}{17}}-3}{3}$	$\frac{3-\sqrt{\frac{89}{17}}}{3}$	$\frac{2\sqrt{\frac{89}{17}}}{3}$	$3-\sqrt{\frac{89}{17}}, 3-\sqrt{\frac{89}{17}}$
32	$(0.4927, 0.5714)$	0.5129	0.5326	0.2386	1.523	$0.7159, 0.6962$
33	$\left(\frac{261}{512}, \frac{309}{512}\right) \simeq (0.5098, 0.6035)$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{2}$	$1, \frac{3}{4}, \frac{3}{4}, \frac{3}{4}$
34	$\left(\frac{553\sqrt{553}-7047}{11616}, \frac{575\sqrt{553}-6453}{11616}\right)$ $\simeq (0.5129, 0.6085)$	$\frac{\sqrt{553}-6}{33}$	$\frac{\sqrt{553}-6}{33}$	$\frac{39-\sqrt{553}}{66}$	$\frac{\sqrt{553}+27}{33}$	$\frac{39-\sqrt{553}}{22}, \frac{39-\sqrt{553}}{22}, \frac{39-\sqrt{553}}{22}$

34 good fixed points (blue dots) + 36 "bad" fixed points (yellow dots)

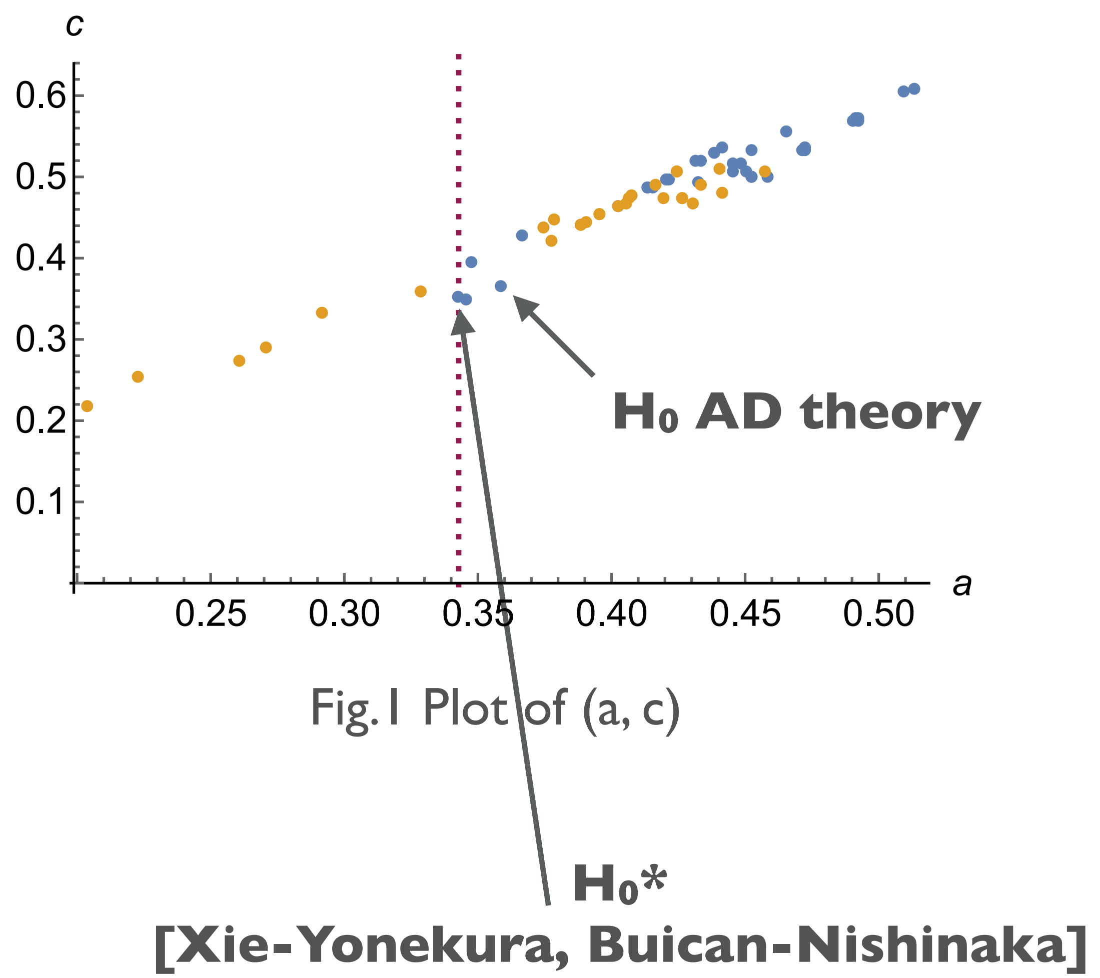


Fig. 1 Plot of (a, c)

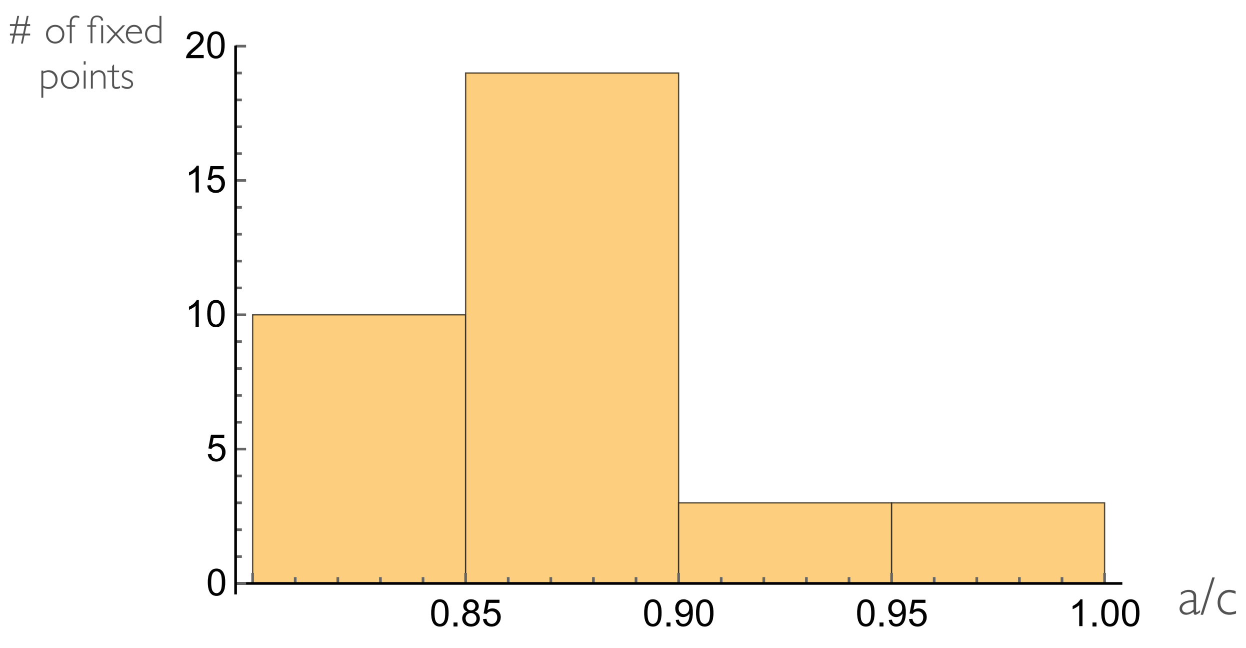


Fig. 2 Histogram of the ratio a/c of the 35 fixed points.

- **H_0^*, minimal a:** $W = X\mathrm{tr}\phi^2 + \mathrm{tr}\phi q^2 + M\mathrm{tr}\phi q'^2 + M^2$

  There is no global U(1) symmetry other than $U(1)_R$, the central charges

$$a_{H_0^*} = \frac{263}{768} \simeq 0.3422, \quad c_{H_0^*} = \frac{261}{768} \simeq 0.3529.$$

  which are the same as those studied by **[Xie-Yonekura, Buican-Nishinaka]**

- **T_0, minimal c:** $W = X\mathrm{tr}\phi^2 + \mathrm{tr}\phi q^2$

  There is a global U(1) symmetry and the central charges are

$$a_{T_0} = \frac{81108 + 1465\sqrt{1465}}{397488} \simeq 0.3451, \quad c_{T_0} = \frac{29088 + 1051\sqrt{1465}}{198744} \simeq 0.3488.$$

  Also, minimal a for SCFTs with global U(1). [Benvenuti]

Both theories have the scalar operator $\mathcal{O}$ with the lowest dimension **satisfying the relation $\mathcal{O}^2 \sim 0$**. cf. **[Poland-Stergiou]**

Conclusions and discussions

- We considered two different deformation procedures which produce various fixed points including N=2 susy enhanced ones.
- What is the precise conditions for susy enhancement?
- Why susy enhancement??
- Localization computations **[Fredrickson-Pei-Yan-Ye, Gukov, Fluder-Song]**
- Toward minimal N=1 SCFT **[Poland-Stergiou]**
- Holographic dual of the RG flow with the enhanced susy.
- string/M-theory realization? **[Giacomelli, Carta-Giacomelli-Savelli]**